Assignment-1 (NCERT 11.1- Intersection Of Two Lines , Shortest Distance Between Two Lines)

1. Find the length and foot of perpendicular draw from the point (2, -1, 5) to line $\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}$.
2. Find the point on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance $3\sqrt{2}$ from the point (1, 2, 3).
3. Show that the lines $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5}$ and $\frac{x+2}{4} = \frac{y-1}{3} = \frac{z+1}{-2}$ do not intersect.
4. Find the equations of two lines through the origin which intersect the line $\frac{x-3}{2} = \frac{y-3}{1} = \frac{z}{1}$ at angles of $\frac{\pi}{3}$.
5. Vertices B and C of Δ ABC lie along the line $\frac{x+2}{2} = \frac{y-1}{1} = \frac{z-0}{4}$. Find the area of the triangle given that A has coordinates (1, -1, 2) and line segment BC has length 5.
6. A line makes angles α, β, γ, δ *with the four diagonals of a cube, prove that* $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + \cos^2 \delta = \frac{4}{3}$.
7. The points A (4, 5, 10) B (2, 3, 4) and C (1, 2, -1) are there vertices of a parallelogram ABCD. Find the vector equations of the sides AB and BC and also find the coordinates of point D.
8. A line with direction ratios < 2, 1, 2 > meets each of the lines given by the equations x = y + a = z and x + a = 2y = 2z. Find the coordinates of each of these points of intersection.
9. The Cartesian equations of a line are 6x – 2 = 3y + 1 = 2z – 2. Find its direction ratios and also find vector equation of the line.
10. Find the equation of the line passing through P(-1, 3, -2) and perpendicular to the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$.
11. Find the condition that the lines x = ay +b, z = c y + d and x = a' y + b', z = c' y + d' may be perpendicular to each other.
12. Find the point on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance 5 units from the point P (1, 2, 3).
13. Find the foot of perpendicular from the point P(1,2,3) on the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$. Also obtain the equation and the length of perpendicular.
14. Find the equation of the line passing which intersect the lines $\frac{x-1}{2} = \frac{y-2}{2} = \frac{z-3}{4}$ and $\frac{x+2}{1} = \frac{y-3}{2} = \frac{z+1}{4}$ and passes through the point(1,1,1).
15. Show that the angles between the diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$.
16. Find the equations of the lines intersecting the lines $\frac{x}{1} = \frac{y+a}{1} = \frac{z}{1}$ and parallel to the lines $\frac{x-a}{2} = \frac{y+a}{1} = \frac{z-2a}{2}$
17. Find the shortest distance between the following pairs of lines and hence write whether the lines are intersecting or not $\frac{x-1}{2} = \frac{y+1}{3} = z,\ \frac{x+1}{5} = \frac{y-2}{1}$; z= 2
18. Find the angles between the lines whose distance cosine are given by the equations 3l + m + 5n = 0, 6mn - 2nl + 5lm = 0
19. Find the foot of perpendicular drawn from the point $2\hat{\imath} - \hat{\jmath} + 5\hat{k}$ to the line $\vec{r} = (11\hat{\imath} - 2\hat{\jmath} - 8\hat{k}) + \lambda(10\hat{\imath} - 4\hat{\jmath} - 11\hat{k})$
20. Find the points on the line through the points A(1,2,3) and B(3,5,9) at a distance of 14 units from te mid-point of segment AB.

ANSWERS

1. Length= $\sqrt{14}$ units , foot of perpendicular is (1,2,3)
2. Required point on the line is (-2,-1,3) or $\left(\frac{56}{17}, \frac{43}{17}, \frac{11}{17}\right)$
4. Required line are : $\frac{x}{1} = \frac{y}{2} = \frac{z}{-1}$ and $\frac{x}{-1} = \frac{y}{1} = \frac{z}{-2}$
5. $\sqrt{\frac{1775}{28}}$ sq. units
7. Equation of line AB : $\vec{b} = (4\hat{\imath} + 5\hat{\jmath} + 10\hat{k}) + \lambda(2\hat{\imath} + 2\hat{\jmath} + 6\hat{k})$
   Equation of line BC : $\vec{d} = (2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}) + \mu(\hat{\imath} + \hat{\jmath} + 5\hat{k})$
   Coordinates of D are (3,4,5)
8. the points are (3a, 2a, 3a) and Q (a, a, a)
9. $\vec{r} = (\frac{1}{3}\hat{\imath} - \frac{1}{3}\hat{\jmath} + \hat{k}) + \lambda(2\hat{\imath} + 2\hat{\jmath} + 6\hat{k})$
10. Required Equation is : $\frac{x+1}{4} = \frac{y-3}{-14} = \frac{z+2}{8}$ or $\frac{x+1}{2} = \frac{y-3}{-7} = \frac{z+2}{4}$
11. $a.a' + 1.1 + c.c' = 0$ iff $aa' + cc' + 1 = 0$
12. the required points are (-2,-1,3) for μ=0 and (4,3,7) for μ= 2
13. length of perpendicular= 7 units
14. Equation of line is $\frac{x-1}{3} = \frac{y-1}{10} = \frac{z-1}{17}$
16. Equation of line is $\frac{x-a}{2} = \frac{y-a}{1} = \frac{z-a}{2}$
17. $\frac{9}{\sqrt{195}}$ units
18. $\theta = \cos^{-1}\left(\frac{1}{6}\right)$
19. $\sqrt{14}$
20. The Required points are (6, $\frac{19}{2}$, 18) And (-2 ,-$\frac{5}{2}$, -6)

Assignment -2 (NCERT Exercise 11.2 –Equation Of A Plane In Normal Form , Intercept Of The Equation)

1. Find the equation of the plane passing through the point (2,1,0),(3,-2,-2) and (3,1,7)

2. A Plane meets the coordinate axes in A,B,C such that the centroid of triangle ABC is the point (p, q, r). show that the equation of the plane is $\frac{x}{p}+\frac{y}{q}+\frac{z}{r}=3$
3. Find the equation of the perpendicular drawn from the point (1, -2, 3) to the plane 2x - 3y + 4z +9=0. Also , Find the coordinates of the foot of the perpendicular.

4. Find the coordinates of the points where the line through (3,-4,-5) and (2,-3,1) crosses the plane determined by points A (1,2,3), B(2,2,1) and C(-1,3,6).

5. If from a point P(a, b, c) perpendicular PA and PB are drawn to yz and zx-planes, find the vector equation of the plane OAB
6. Reduce the equation of the plane 3x + 4y-z +7=0 in the normal form and hence find its distance from origin.
7. Find the distance of the point A(-2,3,-4) from the line $\frac{x+2}{3}=\frac{2y+3}{4}=\frac{3z+4}{5}$ measured parallel to the plane 4x + 12y - 3z + 1 = 0
8. A Variable plane moves in such a way that the sum of the reciprocals of its intercepts on the three coordinates axes is constant. Show that the plane passes through a fixed point.

9. Find the length and the foot of perpendicular from the point(1,1,2) to the plane 2x – 2y + 4z + 5 = 0
10. The foot of perpendicular drawn from the origin to the plane is(4,-2,-5). Find the equation of the plane.

11. The position vectors of two points A and B are $3\hat{\imath}+\hat{\jmath}+2\hat{k}$ and $\hat{\imath}-2\hat{\jmath}-4\hat{k}$ respectively. Find the vector equation of the plane passing through B and perpendicular to the vector $\overrightarrow{AB}$.
12. Find the distance of the point (1,-2,3) from the plane x – y + z =5 measured along a line parallel to $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$

13. Find the distance of the point (-1,-5,-10) from the point of intersection of the line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$ and the plane x – y + z = 5
14. Find the equation of the plane passing through the point (-1,2,1) and perpendicular to the line joining the points(-3,1,2) and (2, 3,4) . Find also the perpendicular distance of the origin from this plane

15. A Vector $\vec{n}$ of magnitude 8 units is inclined to the x- axis at 45°, y – axis at 60° and an acute angle with z – axis . if a plane passes through a point($\sqrt{2}$,-1,1) and is normal to $\vec{n}$. Find its equation in vector form.

ANSWERS

1. 21x + 9y- 3z- 51 = 0
3. Equation of perpendicular is $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z-3}{4}$ = λ and Coordinates are(-1,1,-1)
4. Coordinates are (1 , -2, 7)
5. Equation of plane is : $\vec{r}.\,(\,bc\hat{\imath} + \;ca\hat{\jmath} - ab\hat{k})$ =0
6. $\frac{7}{\sqrt{26}}$.
7. $\frac{17}{2}$
9. $\frac{13}{12}\sqrt{6}$ units
10. $4x - 2y - 5z = 45$ is the required equation of the plane
11. $\vec{r}.\,(2\,\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$ =0
12. 1 units
13. 13
14. $\frac{7}{\sqrt{33}}$
15. $\vec{r}.\,(\sqrt{2}\,\hat{\imath} + \hat{\jmath} + \hat{k})$ =2

Assignments-3 (NCERT Exercise 11.3 –Angle Between Two Planes,Coplanarity Of Two Lines)

1. If the line $\vec{r}$ = $(\hat{\imath} - 2\hat{\jmath} + \hat{k})$ +λ $(2\hat{\imath} + \hat{\jmath} + 2\hat{k})$ is parallel to the plane $\vec{r}.(3\hat{\imath} - 2\hat{\jmath} + m\hat{k})$ =14 , find the value of m
2. Show that the line $\vec{r}$ = $(\hat{\imath} + \hat{\jmath})$ + λ $(2\hat{\imath} + \hat{\jmath} + 4\hat{k})$ lies on the plane $\vec{r}.(\hat{\imath} + 2\hat{\jmath} - \hat{k})$ = 3
3. Find the angle between the line $\frac{x+1}{3} = \frac{y-1}{2} = \frac{z-2}{4}$ and the plane 2x + y – 3z = 0
4. Find the length and foot of the perpendicular from the point (7,14,5) to the plane 2x + 4y – z = 2
5. Find the equation of the plane passing through the intersection of the planes 2x – 3y + z – 4 = 0 and x – y + z + 1 = 0 and perpendicular to the plane x + 2y -3z + 6 = 0
6. Find the equation of the plane passing through the intersection of the planes $\vec{r}.(\hat{\imath} + \hat{\jmath} + \hat{k})$ = 1 and $\vec{r}.(2\hat{\imath} + 3\hat{\jmath} - \hat{k})$ + 4 = 0 and parallel to x- axis.
7. Find the equation of the plane passing through the point $2\hat{\imath} - \hat{k}$ and parallel to the lines $\frac{x}{-3} = \frac{y-2}{4} = z + 1$ and x - 4 = $\frac{1-y}{2}$ = 2z
8. Find the vector equation of the plane passing through the point $\hat{\imath} + \hat{\jmath} - 2\hat{k}$, $2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $(\hat{\imath} + 2\hat{\jmath} + \hat{k})$
9. Find the equation in Cartesian form as well as vector equation of a plane passing through the point (3,-3,1) and normal to the line passing through the points(3,4,-1) and(2,-1,5)
10. Find the equation of the plane through the line of intersection of $\vec{r}.(2\hat{\imath} - 3\hat{\jmath} + 4\hat{k})$ = 1 and $\vec{r}.(\hat{\imath} - \hat{\jmath})$+ 4 =0 and perpendicular to $\vec{r}.(2\hat{\imath} - \hat{\jmath} + \hat{k})$+ 8 = 0
11. Find the equation of the plane passing through the line of intersection of the planes x + 2y +3z -5 = 0 and 3x -2y –z + 1 = 0 and cutting off equal intercepts on x-axis and z-axis
12. If from a point P(a, b, c) perpendiculars PA and PB are drawn to yz and zx planes, then find the vector equation of the plane OAB.
13. Find the plane passing through (4,-1,2) and parallel to the lines $\frac{x+2}{3} = \frac{y-2}{-1} = \frac{z+1}{2}$ and $\frac{x-2}{1} = \frac{y-3}{2} = \frac{z-4}{3}$
14. Find the direction ratio of the normal to the plane passing through the point (2,1,3) and the line of intersection of the planes x + 2y + z = 3 and 2x - y- z = 5
15. Find the equation of the two planes passing through the points (0,4,-3) and (6,-4,3) if the sum of their intercepts on the three axe is zero .
16. A straight line passes through the point (2,-1,-1). It is parallel to the plane 4x+y+z+2=0 and is perpendicular to the line $\frac{x}{1} = \frac{y}{-2} = \frac{z-5}{1}$. find its equation.
17. A variable plane is at a constant distance p from the origin and meets the coordinates axes in A, B, C . Show that the locus of the centroid of the tetrahedron OABC is $x^{-2} + y^{-2} + z^{-2} = 16\,p^{-2}$
18. Find the vector and Cartesian equation of the plane containing the two lines $\vec{r}$ = $(2\hat{\imath} + \hat{\jmath} - 3\hat{k})$ + l $(\hat{\imath} + 2\hat{\jmath} + 5\hat{k})$ and $\vec{r}$ = $(3\hat{\imath} + 3\hat{\jmath} + 2\hat{k})$ + m $(3\hat{\imath} - 2\hat{\jmath} + 5\hat{k})$
19. Find the distance of the point P$(\hat{\imath} + \hat{\jmath} + \hat{k})$ from the plane through the points A$(2\hat{\imath} + \hat{\jmath} + \hat{k})$, B $(\hat{\imath} + 2\hat{\jmath} + \hat{k})$ and C$(\hat{\imath} + \hat{\jmath} + 2\hat{k})$. Also , Find the position vector of the foot of perpendicular from P on this plane.

ANSWERS

1. m = -2
3. θ = $\sin^{-1}\left(\frac{-4}{\sqrt{406}}\right)$
4. $3\sqrt{21}$
5. x – 5y – 3z – 23 = 0 is the required equation of the plane
6. $\vec{r}\,.(-\hat{j}+3\hat{k})$ = 6 is the required equation of the plane
7. $\vec{r}$. ($8\hat{\imath}+5\hat{j}+4\hat{k}$)-12 = 0 is the required vector equation
8. $\vec{r}$. ($9\hat{\imath}+3\hat{j}-\hat{k}$) = 14
9. -x-5y + 6z =18 0r x + 5y -6z + 18 = 0 is the required Cartesian form
10. $\vec{r}$. ($-5\hat{\imath}+2\hat{j}+12\hat{k}$) = 47
11. $5x+2y+5z$–9=0
12. Equation of plane is : $\vec{r}$. ($bc\hat{\imath}+\ ca\hat{j}-ab\hat{k}$) =0
13. x + y –z – 1 = 0
14. direction ratio of normal to this plane are proportional to 13,6,1
15. 6x+3y-2z = 18 or 2x- 3y – 6z = 6
16. $\frac{x-2}{1}=\frac{y+1}{-1}=\frac{z+1}{-3}$
18. Vector equation of plane is $\vec{r}$. ($10\hat{\imath}+5\hat{j}-4\hat{k}$) = 37
and Cartesian equation is 10x + 5y – 4z = 37
19. Distance of the point P =$\frac{1}{\sqrt{3}}$
Position Vector of N are $\frac{4}{3}$ $(\hat{\imath}+\hat{j}+\hat{k}\,)$